Interpretability Indices for Hierarchical Fuzzy Systems

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***Abstract*—Hierarchical fuzzy systems (HFSs) have been shown to have the potential to improve interpretability in fuzzy logic systems (FLSs). In recent years, a variety of indices has been proposed to measure the interpretability of FLSs such as the *Nauck index* and *Fuzzy index*. However, interpretability indices associated with HFSs have not so far been discussed. The structure of HFSs, with multiple layers, subsystems, and varied topologies, is the main challenge in constructing interpretability indices for HFSs. Thus, the comparison of interpretability between FLSs and HFSs—even at the index level—is still the subject to an open discussion. This paper begins to address these challenges by introducing extensions to the FLS *Nauck* and *Fuzzy* interpretability indices for HFSs. Using the proposed indices, we explore the concept of interpretability in relation to the different structures in FLSs and HFSs. Initial experiments on benchmark datasets show that based on the proposed indices, HFSs with equivalent function to FLSs produce higher indices, i.e. are more interpretable than their corresponding FLSs.**

I. Introduction

One of the strengths of Fuzzy Logic Systems (FLSs) is their interpretability [1], particularly in applications such as knowledge extraction and decision support [2], [3]. However, key challenges remain around FLS interpretability, including the *curse of dimensionality*: the number of required rules commonly increases exponentially with the number of input variables [4]. This challenge is also known as rule explosion which may reduce the transparency and interpretability of FLSs [5]. One effective way to deal with this problem is through the use of a special type of FLS, namely hierarchical fuzzy systems (HFSs) [6]–[11].

HFSs were introduced by Raju [6] as an approach to overcome the *curse of dimensionality* which arises in conventional FLSs. In HFSs, the original FLSs are decomposed into a series of low-dimensional FLSs—fuzzy logic subsystems (see Section II-B). Moreover, the rules in HFSs commonly have antecedents with fewer variables than the rules in FLSs with equivalent function, since the number of input variables of each subsystem is lower [12], [13]. Thus, HFSs tend to reduce rule explosion, thus minimizing complexity, and improving model interpretability. So far, the potential of HFSs to support good interpretability in FLSs has not been explored in detail, with only a small number of works considering it [5], [14]–[17].

In this paper, an extension of the two most common FLS interpretability indices, namely *Nauck index* and *Fuzzy index*, to HFSs is proposed, with a specific focus on the complex structure of HFSs such as having multiple layers, subsystems and varied topologies.

This paper is organized as follows; Section II discusses background on interpretability in FLSs and HFSs, in particular, the common interpretability indices: the *Nauck* and *Fuzzy index*, as well as providing an overview of the challenges in developing interpretability indices for HFSs. Section III introduces the extension of the interpretability indices from FLSs to HFSs, demonstrated with synthetic examples. Finally, Section IV, V and VI present experiments, results, conclusion and future works.

II. Background

A. Interpretability of FLS

Interpretability indicates how easily an FLS can be understood by human beings [18]. In recent years, the interest of researchers in obtaining more interpretable fuzzy models has increased. However, the choice of an appropriate interpretability measure is still an open discussion due to its subjective nature and the large amount of factors involved. Substantial research on interpretability measures [19]–[27] proposed interpretability indices for FLSs. The most common interpretabilty indices are *Nauck index* and *Fuzzy index*.

1) Nauck index:

Nauck index is a numerical index introduced by Nauck [21] in order to measure fuzzy rule-based classification systems. In this paper, we are using it as a useful index for assessing the interpretability of FLSs. It is computed as the product of three terms:

$$\textit{Nauck index} = comp \times \overline{cov} \times \overline{part}. \tag{1}$$

- $comp$ represents the complexity of FLSs measured as the number membership functions (MFs) of output variables divided by the number of input variables in FLSs rules. It is computed as

$$comp = m / \sum_{i=1}^{r} n_i \tag{2}$$

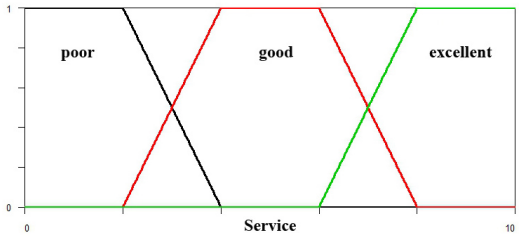

Fig. 1. Membership function of *service* that shows complete coverage.

where m is the number MFs of output variables, r is the number of rules and n_i is the number of input variables used in the ith rule.

- cov is the coverage degree of the fuzzy partition. If X_i is the domain of ith input variable partitioned by p_i MFs $\{\mu_i^{(1)},\ldots,\mu_i^{(p_i)}\}$, then cov is computed as

$$cov_i = \frac{\int_{X_i} \hat{h_i}(x)\,\mathrm{d}x}{N_i}$$
$$\hat{h_i}(x) = \begin{cases} h_i(x) & \text{if} \quad 0 < h_i(x) < 1 \\ \frac{p_i - h_i(x)}{p_i - 1}, & \text{otherwise} \end{cases} \tag{3}$$
$$h_i(x) = \sum_{k=1}^{p_i} \mu_i^{(k)}(x),$$

where $h_i(x)$ is the total MFs of ith input variable with $N_i = \int_{X_i}$ for continuous domains. The integral in (3) is replaced by a sum for discrete finite domains with $N_i = |X|$. Then, $\overline{cov} = \sum_{i=1}^{r} cov_i/n_i$, denotes the average normalized coverage for all input variables.

- $part$ stands for the partition index which is computed as the inverse of the number of MFs minus one for each input variable;

$$part_i = \frac{1}{p_i - 1}, \tag{4}$$

where p_i is the number of MFs in the ith input variable. Then, $\overline{part} = \sum_{i=1}^{r} part_i/n_i$, denotes the average normalized partition index for all input variables.

An FLS model is said to be less interpretable when its *Nauck index* is closer to 0 and more interpretable when *Nauck index* is closer to 1. To illustrate how the interpretability index is generated by *Nauck index*, we consider the Waiter-Tipping example [28] which is frequently used as a benchmark. The rule base of the example is as follows:

IF *service* is *poor* THEN *tip* is *cheap*,
IF *service* is *good* THEN *tip* is *average*,
IF *service* is *excellent* THEN *tip* is *generous*.

The *Nauck index* can be generated as:

$$comp = \frac{3}{1+1+1} = 1$$

$$\overline{cov} = 1 \text{(i.e., as shown in Fig. 1)}$$

$$\overline{part} = \frac{1}{3-1} \text{(only one input variable is used)}$$

TABLE I
FUZZY INDEX OF TIPPER EXAMPLE

Total number of rules	:	3
Total number of inputs in all rules	:	3
Rule with 1 input	:	3
Rule with 2 inputs	:	0
Rule with 3 inputs or more	:	0
Average linguistic terms define by inputs	:	3
Fuzzy index	:	0.647

$$\textit{Nauck index} = comp \times \overline{cov} \times \overline{part} = 0.5.$$

2) Fuzzy index:

As discussed in [22], the *Fuzzy index*, which is inspired by *Nauck's index* is proposed in interpretability assessment. Six variables are taken as input of a fuzzy system namely; (i) total number of rules, (ii) total number of input variables in all the rules, (iii) number of rules which use one input variable, (iv) number of rules which use two input variables, (v) number of rules which use three or more input variables, (vi) average number of linguistic terms defined for each input variable.

The *Fuzzy index* itself is computed as the result of a hierarchical FLS (see Section II-B) which consists of four linked knowledge bases, namely: RB1, RB2, RB3 and RB4. RB1 gives an estimation of the rule base dimension based on the total number of rules and input variables. At the same time, the rule base complexity is evaluated at RB2 according to the number of input variables used by the rules. Then, RB3 combines rule base dimension and complexity, and as a result it yields a rule base interpretability index. Lastly, RB4 integrates the rule base interpretability index and the average number of linguistic terms defined by input variable.

A *Fuzzy index* closer to 0 implies that the FLS model is less interpretable. Meanwhile, a *Fuzzy index* closer to 1 implies higher interpretabality. To show how this index is generated, we use the same Waiter-Tipping example rule base. The *Fuzzy index* calculation is summarized in Table I.

B. Hierarchical Fuzzy Systems

Definition 1 : HFSs are characterized by composing the input variables into a collection of low-dimensional FLSs—fuzzy logic subsystems [6], [7]. HFSs can be illustrated as a cascade structure where the output of each layer is considered as an input to the following layer as shown in Fig. 3.

Definition 2 : HFSs can be viewed as a functional decomposition of FLSs [29]. For instance, FLS and HFS as shown in Fig. 2 and Fig. 3 respectively can be presented, from a functional point of view as

$$y = F(x_1, x_2, x_3) \implies y = f_2(x_3, f_1(x_2, x_1)).$$

A system that goes from one layer as shown in Fig. 2 to two layers as in Fig. 3 has fewer rules than the one in one layer. The most extreme reduction of rules will be if the structure of HFS has two input variables for each low dimensional FLS and has $(n-1)$ layers [6]. If we define m fuzzy sets for each input variable, including the intermediate output variables $y_1, ..., y_{n-2}$, the total number of rules (R) is a linear function

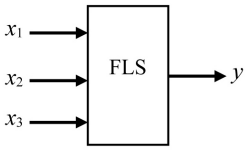

Fig. 2. Fuzzy Logic System

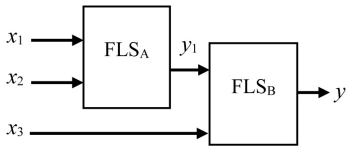

Fig. 3. Hierarchical Fuzzy System

[8] of the number of input variables n and can be expressed as:

$$R = (n-1)m^2. \tag{5}$$

In conventional FLSs, the number of rules increases exponentially with the increase in the number of input variables [6], [30]. Supposed there are n input variables and m fuzzy sets for each input variable, then m^n rules are needed to construct a complete fuzzy system with fully specified rule base (using the "AND" logical connective). For example, Fig. 2 and Fig. 3 show a FLS and HFS with 3 input variables ($n = 3$) and, assuming that 3 fuzzy sets ($m = 3$) are defined for each input variable, the total number of rules for this FLS is $m^n = 3^3 = 27$ whereas for the HFS, the total number of rules is $(n-1)m^2 = (3-1)3^2 = 18$ i.e. it is clear that the total number of rules is always lower or equal when employing a HFS.

Previous research has shown that HFSs have been used to improve interpretability [5], [14]–[17]. However, to the authors' knowledge, no one has investigated how interpretability can similarly be measured using indices in HFSs. In this paper, an extension to the interpretability indices from FLSs to HFSs is proposed. In doing so, the following challenges of HFSs will be explored:

1) Aggregation:

The first challenge is the choice of aggregation technique to aggregate the information or values provided at different subsystems in HFSs. For example, assume that we obtain two values of an interpretability index which are I_{FLS_A} and I_{FLS_B} calculated for subsystem FLS_A and FLS_B as shown in Fig. 3. In order to calculate the interpretability index in HFSs, we need to determine the method to aggregate individual values such as I_{FLS_A} and I_{FLS_B}. However, to determine the aggregation method in HFSs, we have to consider two situations. The first is the aggregation process between the layers and the second is the aggregation process in the same layer.

2) Design Decision on Topology & Layering:

As mentioned in Definition 1, HFSs are produced by decomposing the input variables in FLSs into multiple low-dimensional FLSs. By doing this, several *layers* are produced in HFSs. Based on the same input variables, HFSs may be produced using different topologies, e.g., serial and parallel HFS [12]. The parallel HFS can have more than one low-dimensional FLS per layer, while serial HFSs use strictly one FLS per layer as shown in Fig. 4 and Fig. 5 respectively. Thus, these topologies commonly have a different number of layers. For example, Fig. 4 and Fig. 5 show two different HFS topologies using the same four input variables namely

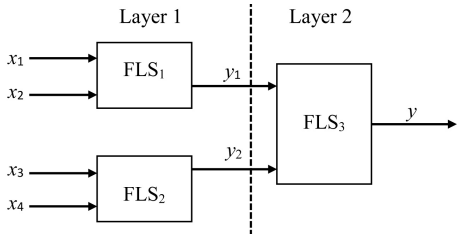

Fig. 4. Parallel HFS with 2 layers.

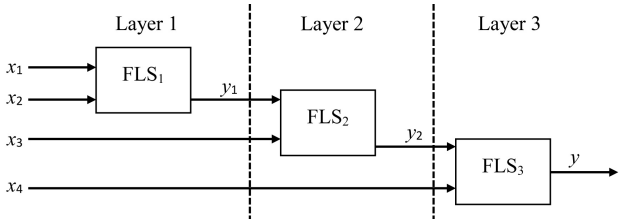

Fig. 5. Serial HFS with 3 layers.

x_1, x_2, x_3 and x_4. Both HFSs topologies use the same number of subsystems, but with different numbers of layers in their structure.

III. Interpretability Indices for HFSs

A. Motivation

In Section II-B, some challenges in proposing the interpretability indices in HFSs were presented. From these, several questions arise, including: "What is the best aggregation technique across and within layers?", and "How should new interpretability indices deal with different topologies?". Taking these questions into consideration, we propose an initial solution by extending both the *Nauck* and *Fuzzy* indices from their FLS form to HFSs.

B. The Proposed Interpretability Indices for HFSs

As an initial step, in this paper, we extend the *Nauck* and *Fuzzy* indices by proposing a general extension for such indices from FLSs to HFSs. We refer to the general extension as *HFSi*. By considering the challenges that have been discussed in Section II-B, we propose an aggregation strategy capturing the HFS topology with *layer-weights* indicating the weight associated with each layer. In this paper, we only focus on HFSs with one output variable and no cyclic connections between internal FLSs. In this context, *HFSi* is computed as follows:

$$\textit{HFSi} = \sum_{i=1}^{n} (l_i \sum_{j=1}^{m_i} E_{ij}/m_i), \tag{6}$$

where E_{ij} is for example the *Nauck* (N) or *Fuzzy* (F) *index* of a subsystem *j* at layer *i*, l_i is the associated weight to the layer *i* of the HFSs, m_i is the number of subsystems located at the layer *i*, and n is the number of layers.

Layer-weights, l_i are associated to each subsystem according to their layer. In this paper, we propose a *layer-weighting* which has the following properties:

(i) $\sum_{i=1}^{n} l_i = 1$, i.e. the summation value of all *layer-weights* l_i should be equal to 1 regardless the number of layers n.

(ii) $l_1 > l_2 > ... > l_n$, i.e. the *layer-weights*, l_i are arranged in descending order according to the challenges in Section II-B. As discussed in [6], [7], most HFSs structure is formed by having the most influential input variables to the first layer of the hierarchy, the next most important inputs to the second layer, and so on.

In order to achieve the above properties, l_i is proposed as:

$$l_i = \frac{2(n-i+1)}{n(n+1)}, \quad i = 1, ..., n. \tag{7}$$

A HFS model is less interpretable when the *HFSi* is close to 0 and more interpretable when *HFSi* is close to 1. In the following section, we will explore the features of the *HFSi* for the cases of the *Nauck* and *Fuzzy* interpretability indices by using a real-world example.

IV. Experiments

As a demonstration, we have used an example of a seesaw control application which enables the systematic comparison between conventional FLS and HFS in order to solve the problem of balancing in seesaw. The involved parameters of seesaw are the distance of the cart (x_1), the angle that the wedge makes with the vertical line (x_2), the height of the wedge (x_3) and center of mass of the wedge (x_4) as in [30] .

A. Seesaw control application

The seesaw control example from Lee et al. [30] is used. As reported in their study, this system has 4 input variables ($n = 4$) namely x_1, x_2, x_3 and x_4 with 3 fuzzy sets ($m = 3$): *pb*, *ze* and *nb* are defined for each input variable. The total number of rules for this standard FLS of a seesaw control is obtained by $m^n = 3^4 = 81$ rules and can be illustrated as follows:

- IF x_1 is *pb* AND x_2 is *pb* AND x_3 is *pb* AND x_4 is *pb* THEN y is *nb*,
- IF x_1 is *pb* AND x_2 is *pb* AND x_3 is *pb* AND x_4 is *ze* THEN y is *nb*,
- IF x_1 is *pb* AND x_2 *pb* AND x_3 is *pb* AND x_4 is *nb* THEN y is *nm*,

  $\vdots$
- IF x_1 is *nb* AND x_2 is *pb* AND x_3 is *nb* AND x_4 is *nb* THEN y is *pb*.

In this study, two types of HFS topologies (parallel and serial) were chosen and developed using the Fuzzy Toolbox in the R programming language [31].

Table II shows the interpretability measurements of the standard FLS and all subsystems of the HFSs for the seesaw example for both the *Nauck* and *Fuzzy* indices. The aim of this experiment is to explore how the interpretability indices behave for several HFS toplogies. Then, the overall results of the interpretability indices are compared with those of the conventional FLS.

1) *Parallel implementation of the Seesaw Control*
Figure 6 shows the interpretability indices measured for each subsystem in the parallel HFS implementation of the seesaw control by using the *Nauck* and *Fuzzy* indices respectively. Due to the limited space in this paper, we only provide one sample structure of the rules for this parallel HFS. The rules for subsystems FLS_1, FLS_2, and FLS_3 can be illustrated as follows:
For FLS_1 :

- IF x_1 is *pb* AND x_2 is *pb* THEN y_1 is *pb*,
- IF x_1 is *pb* AND x_2 is *ze* THEN y_1 is *pb*,

  $\vdots$
- IF x_1 is *nb* AND x_2 is *nb* THEN y_1 is *nb*.

For FLS_2 :

- IF x_3 is *pb* AND x_4 is *pb* THEN y_2 is *pb*,
- IF x_3 is *pb* AND x_4 is *ze* THEN y_2 is *pb*,

  $\vdots$
- IF x_3 is *nb* AND x_4 is *nb* THEN y_2 is *nb*.

For FLS_3 :

- IF y_1 is *pb* AND y_2 is *pb* THEN y is *pb*,
- IF y_1 is *pb* AND y_2 is *ze* THEN y is *pm*,

  $\vdots$
- IF y_1 is *nb* AND y_2 is *nb* THEN y is *nb*.

The values of *Nauck* and *Fuzzy* indices for the three subsystems are $\text{N}_1 = 0.083$, $\text{N}_2 = 0.083$, $\text{N}_3 = 0.194$, $\text{F}_1 = 0.493$, $\text{F}_2 = 0.493$ and $\text{F}_3 = 0.605$ (the details are shown in Table II). There are two layers in this HFS ($n = 2$). The values of the layer-weights using equation (7) are $l_1 = 0.667$ and $l_2 = 0.333$. According to equation (6), the suggested interpretability indices of this HFS are:

$$\begin{aligned} \textit{HFSi} &= \sum_{i=1}^{n}(l_i \sum_{j=1}^{m_i} E_{ij}/m_i) \\ &= l_1(N_1 + N_2)/2 + l_2(N_3/1) \\ &= 0.667(0.083 + 0.083)/2 + 0.333(0.194) = 0.120 \\ \textit{HFSi} &= \sum_{i=1}^{n}(l_i \sum_{j=1}^{m_i} E_{ij}/m_i) \\ &= l_1(F_1 + F_2)/2 + l_2(F_3/1) \\ &= 0.667(0.493 + 0.493)/2 + 0.333(0.605) = 0.530 \end{aligned}$$

2) *Serial implementation of the Seesaw Control*
Figure 7 shows the interpretability indices measured for each subsystem in the serial HFS implementation of the seesaw control by using the *Nauck* and *Fuzzy* indices respectively. Similar to the parallel case, the index values obtained for subsystems 1, 2 and 3 are $\text{N}_1 = 0.083$, $\text{N}_2 = 0.083$, $\text{N}_3 = 0.194$, $\text{F}_1 = 0.493$, $\text{F}_2 = 0.493$ and $\text{F}_3 = 0.605$ (the details are shown in Table II). There are three layers in this HFS ($n = 3$). The layer-weights using equation (7) are $l_1 = 0.5$, $l_2 = 0.333$ and $l_3 = 0.167$. According to equation (6), the suggested interpretability

TABLE II
INTERPRETABILITY OF SEESAW CONTROL APPLICATION USING NAUCK AND FUZZY INDEX

Fuzzy Models	*Nauck index*				*Fuzzy index*						
	comp	$\overline{cov}$	$\overline{part}$	*Index*	*NOR*	*NOI*	*Rule 1*	*Rule 2*	*Rule 3*	$\overline{Term}$	*Index*
FLS :											
Seesaw System	0.022	1	0.5	0.011	81	324	0	0	81	3	0.241
Parallel HFS											
Subsystem 1	0.167	1	0.5	0.083	9	18	0	9	0	3	0.493
Subsystem 2	0.167	1	0.5	0.083	9	18	0	9	0	3	0.493
Subsystem 3	0.389	1	0.5	0.194	9	18	0	9	0	3	0.605
Overall				0.120							0.530
Serial HFS											
Subsystem1	0.167	1	0.5	0.083	9	18	0	9	0	3	0.493
Subsystem 2	0.167	1	0.5	0.083	9	18	0	9	0	3	0.493
Subsystem 3	0.389	1	0.5	0.194	9	18	0	9	0	3	0.605
Overall				0.102							0.512

**NOR* = Number of rules, *NOI* = Number of input in all rules, *Rule 1* = Rule with 1 input, *Rule 2* = Rule with 2 inputs, *Rule 3* = Rule with 3 inputs or more, $\overline{Term}$ = Average linguistic terms defined by inputs.

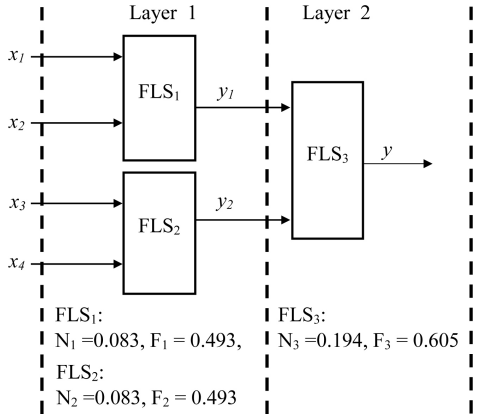

Fig. 6. Interpretability of Parallel HFS using *Nauck index* (N) and *Fuzzy index* (F).

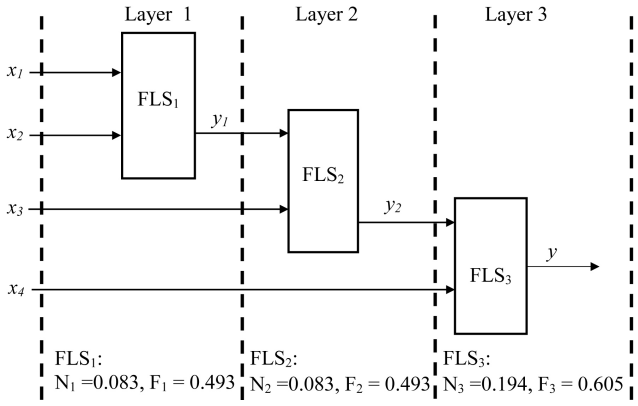

Fig. 7. Interpretability of Serial HFS using *Nauck index* (N) and *Fuzzy index* (F).

indices of this HFS are:

$$HFSi = \sum_{i=1}^{n}(l_i \sum_{j=1}^{m_i} E_{ij}/m_i)$$
$$= l_1((N_1/1) + l_2(N_2/1) + l_3(N_3/1)$$
$$= 0.5(0.083) + 0.333(0.083) + 0.167(0.194) = 0.102$$
$$HFSi = \sum_{i=1}^{n}(l_i \sum_{j=1}^{m_i} E_{ij}/m_i)$$
$$= l_1((F_1/1) + l_2(F_2/1) + l_3(F_3/1)$$
$$= 0.5(0.493) + 0.333(0.493) + 0.167(0.605) = 0.512$$

V. RESULTS AND DISCUSSION

The proposed *HFSi* extension was applied to the example of a seesaw control application for both the *Nauck* and *Fuzzy* indices. The overall interpretability measurements of FLS and HFSs were summarised and are shown in Fig. 8.

For the discussed examples, the computed values of interpretability index in HFSs are larger compared to the FLS regardless of its type (i.e. parallel or serial HFS). As captured

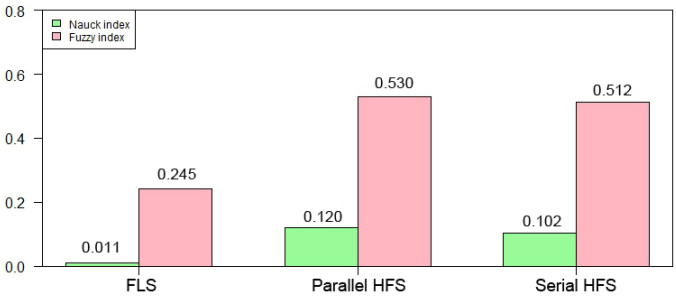

Fig. 8. Interpretability measurement between FLS and HFSs of seesaw control application.

in Fig. 8, the extended *Nauck* indices for parallel serial HFSs are larger than those of the FLS i.e. 0.120 and 0.102 respectively. Figure 8 also shows the same pattern with the extended *Fuzzy* indices with values of 0.530 and 0.512 respectively. We note that in this example of HFSs , the values of the interpretability indices of parallel HFSs are larger than for serial HFSs as shown in both extended interpretability *Nauck* and *Fuzzy* indices of *HFSi*.

The results generated for the extended indices follow in-

tuition in the sense that one expects better interpretability for HFSs than for their FLS counterparts. Clearly, the aggregation strategy adopted in this paper is not unique and a number of different aggregation strategies may be appropriate. In the same context, based on the initial results, it is clear that further work is required to establish ground truth on what the appropriate interpretability index should be, so that it reflects or approximates the interpretability of a system as perceived by an actual human user.

VI. Conclusions

Intuitively, HFSs have the potential to improve interpretability in FLSs. However, interpretability indices associated with HFSs have not so far been discussed. This paper is an initial step. Current research on interpretability are in general aimed at FLSs. On the other hand, HFSs have been introduced in part to facilitate interpretability. In this paper, as a first step to study the interpretability of HFSs, we propose an extension of FLS interpretability indices and apply it to the popular *Nauck* and *Fuzzy* indices.

In initial experiments, the extended indices produced higher results for HFSs than for their equivalent (in functionality) FLS counterparts. This follows intuition as HFSs have been credited with better interpretability compared to FLSs. In the future, a key part of our work will be to conduct experiments with real users to establish the *perceived* interpretability and to use it to validate and inform appropriate interpretability indices. At the moment, beyond the initial motivation of HFSs, it for example is not clear whether a larger number of layers is necessarily easier to interpret than a larger number of single-layer rules.

Future research will also focus on leveraging the information gained from studying interpretability in its human context to inform index aggregation in HFSs. Further, other aspects of interpretability will be explored, including the linguistic meaning of fuzzy sets and the logical complexity of the rules i.e. the impact on interpretability of for example using AND, rather than OR.

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